

Mesotron Decays and the Role of Anomalies

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Puzzles associated with Yukawa's mesotron theory of nuclear interactions led to the discovery of "anomalies" in quantum field theory. I will discuss some of the remarkable consequences of these anomalies in the physics of elementary particles.

In 1935 Hideki Yukawa postulated that nuclear forces were ascribed to a new massive scalar field that coupled to neutrons to protons. To explain the saturation of the nuclear forces, the new mesotrons were required to have a mass of order $200 m_e$, and a coupling a few times larger than that associated with the electric charge. The term mesotron was used to describe a particle of intermediate mass, much heavier than the light electron and much lighter than the neutron and proton, the constituents of the atomic nucleus.

Indeed, in 1937, a new meson of intermediate mass was discovered as the dominant part of the hard component of cosmic rays. It was natural to associate this meson with the field that Yukawa had proposed to explain the nuclear force.

Shoichi Sakata played an important role in the struggle to understand the physics of the new Yukawa mesotrons and their relation to the new mesons seen in cosmic rays.

Early estimates of the lifetime of the mesotron were based on Fermi's theory of β -decay. These estimates, in the range of 10^{-8} to 10^{-7} sec, were considerably shorter than the lifetime observed for the cosmic ray mesons, $\sim 2 \cdot 10^{-6}$ sec. This discrepancy was the focus of several papers by Yukawa, Sakata and collaborators [1, 2].

Sakata also speculated on the lifetime of a neutral mesotron (Neutretto) whose existence was suggested by the charge independence of the nuclear force. Sakata and Tanikawa[3] suggested these neutral mesotrons should decay to photons via the following process:

"First a neutral mesotron is absorbed by a proton which is in the negative energy state and produces a (virtual) pair of a proton and an antiproton. Then this pair disappears with the emission of more than two photons"

Their estimate the lifetime was $\sim 10^{-16}$ sec, remarkably close to the present measured value of the neutral pion lifetime, $(0.84 \pm 0.06) \times 10^{-16}$ sec.

The struggle with the charged mesotron lifetime and the fact that the cosmic ray meson did not interact strongly with nuclei led Tanikawa and Sakata to propose in 1942 that the mesotron and the cosmic ray meson were distinct states [4]. Sakata also suggested that the cosmic ray meson should be a fermion while the Yukawa mesotron is required to be a boson.

These speculations were confirmed in 1947 with the first observation of the charged pion in high altitude cosmic rays [5] and later by pions produced artificially by accelerators [6] in 1948. The neutral pion was also discovered in its two-photon decay mode at accelerators [7] in 1950.

However, theoretical estimates for the lifetime of the neutral pion would lead to another puzzle. New field theory methods were developed to perform the calculation of meson lifetime along the lines originally suggested by Sakata that the neutral meson decays to photons via virtual proton loop. These new methods gave a finite result for a pion with pseudoscalar coupling and a lifetime estimate, $\sim 0.9 \cdot 10^{-16}$ sec. However, the axial-vector coupling remained divergent [8]. The calculations were plagued with divergences and questions of electromagnetic gauge invariance.

A regularization method developed by Pauli and Villars[9] using heavy fermion regulators would permit the gauge invariant calculation of both pseudoscalar and axial-vector matrix elements. This calculation by Steinberger[10] concluded that the pseudoscalar coupling and the axial-vector coupling of the pion to the proton gave finite but different results for the decay amplitude. The field theory equations of motion should have implied that the amplitudes were the same [11] leading to the puzzle concerning the actual prediction for the neutral pion lifetime.

The axial-vector current gained significance with the observation that the strongly interacting particles possess an approximate, hidden chiral symmetry in addition to isospin. Although the nucleon mass appears to break chirality, axial-vector current remains conserved due to the existence of massless pions, PCAC. The pion is a Nambu-Goldstone boson,

$$\langle p' | J_{5\lambda} | p \rangle = \bar{u}(p') \{ g_A \gamma_5 \gamma_\lambda \} u(p) + f (q_\lambda / q^2) \bar{u}(p') \{ g_P \gamma_5 \} u(p), \quad q = p' - p$$

and the axial-vector current remains conserved if $g_A 2M = f g_P$, the Goldberger-Treiman relation[12]. The pion pole compensates for lack of conservation of the axial-vector form factor contribution. The enhanced role of axial-vector current is a reflection of the chiral symmetry of the equations of motion.

An explicit realization of PCAC is provided by the Gell-Mann-Lévy sigma model [13], a renormalizable model of pions, nucleons and a scalar meson. Chiral symmetry is dynamically broken

by a scalar condensate or vev. A term linear in the scalar field can be added to the model which explicitly breaks the chiral symmetry and generates a small pion mass. In this case, the divergence of the axial-vector current is exactly proportional to the pion field. This model is described by the Lagrangian,

$$L_I = - (m/f) \bar{N} (\sigma + i\vec{\tau} \cdot \vec{\pi} \gamma_5) N - \lambda (f^2 - \sigma^2 - \vec{\pi}^2)^2 + f\mu^2 \sigma.$$

In 1969, Bell and Jackiw revisited the calculation of the two-photon matrix element of the axial-vector current in light of PCAC and its explicit realization in the Gell-Mann-Lévy sigma model. Using the Steinberger[10] calculation of the nucleon loops, they found a finite result but that PCAC is explicitly violated for photon matrix elements, the Bell-Jackiw anomaly [14]. They suggest that the fault lies with the use of the Pauli-Villars regulators for the loop calculations and propose an alternative, chiral invariant regulator where the coupling of pions to the heavy regulator fermions grows with the regulator mass. PCAC is restored in the regulated theory but the decay of the neutral pion to photons is highly suppressed. The regulator fields interact according to

$$L_R = - (M_R/f) \bar{\psi}_R (\sigma + i\vec{\tau} \cdot \vec{\pi} \gamma_5) \psi_R.$$

However, the regulator loops do not decouple for large regulator mass and additional nonrenormalizable interactions are generated including a counterterm to the pion-photon-photon vertex. These new interactions will become known as Wess-Zumino terms in nonlinear realizations of dynamical chiral symmetry breaking.

At this same time, Adler studied the properties of the axial-vector current in spinor electrodynamics, a renormalizable model with a chiral symmetry that is only broken by the fermion mass term. Electromagnetic gauge invariance and power counting completely specify matrix elements of the axial vector current. The axial-vector current has an unambiguous anomalous divergence associated with the two photon matrix elements. The field equations for the divergence of the axial current contain a specific additional contribution, the Adler anomaly [15].

$$\partial^\mu \{ \bar{\psi} \gamma_\mu \gamma_5 \psi \} = 2m \{ \bar{\psi} i \gamma_5 \psi \} + \frac{\alpha}{4\pi} Q^2 F^{\mu\nu} \cdot {}^*F_{\mu\nu}$$

The anomaly coefficient is determined by the fundamental charge of the fermion. There is an exact low energy theorem for matrix element of the naïve divergence of the axial-vector current. This low energy theorem is not modified to all orders in perturbation theory, the Adler-Bardeen nonrenormalization theorem [16]. This nonrenormalization theorem identifies the anomaly as a fundamental aspect of quantum field theory and not simply an artifact of a particular perturbative calculation.

The anomalous divergence of the axial-vector isospin current determines the low energy theorem for neutral pion decay to two photons. Since this anomaly can be computed exactly in a specific model, it can be used to identify consistent models of the strong dynamics. The observed lifetime of the pion disagrees with the uncolored quark model [15] and provides evidence for the color triplet quark model [17],

$$\begin{aligned} \text{Nucleon Model (Steinberger):} \quad & 1^2 - 0^2 = 1 \\ \text{Triplet Quark Model:} \quad & (2/3)^2 - (-1/3)^2 = 1/3 \\ \text{Color Triplet Quark Model:} \quad & 3(2/3)^2 - 3(-1/3)^2 = 1 \end{aligned}$$

Anomalies are not restricted to the abelian case considered by Adler, Bell and Jackiw. Anomalies can be generalized through the study of nonabelian currents in field theory. Nonabelian currents can be derived from a general fermion Lagrangian with arbitrary couplings to vector, axial vector, scalar and pseudoscalar fields,

$$L = \bar{\psi} \{ \gamma^\mu V_\mu + \gamma^\mu \gamma_5 A_\mu - \Sigma - i\gamma_5 \Pi \} \psi = \bar{\psi} \{ \Gamma \} \psi$$

Anomalous terms are generated by considering local gauge transformations involving all fields in the effective action which is formally gauge covariant. A precise study shows that all of the anomalous terms can be made to cancel except for those involving certain external vector and axial vector fields with the result [18],

$$\begin{aligned} D(\Lambda_+, \Gamma) &= R(\Gamma i\Lambda_+ - i\Lambda_- \Gamma - \gamma \cdot \partial \Lambda_+, \Gamma) \\ &= \frac{1}{6} \frac{\pi^2}{(2\pi)^4} i \int dz \epsilon_{\mu\nu\sigma\tau} \text{tr} \gamma_5 \{ 2i\Lambda_+ \partial^\mu V_+^\nu \partial^\sigma V_+^\tau - \partial^\mu V_+^\nu V_+^\sigma V_+^\tau \} \end{aligned}$$

where $V_+^v = V^v + A^v \gamma_5$, $\Lambda_+ = \Lambda + \Lambda_5 \gamma_5$.

It is remarkable that the only anomalous divergences that survive are those associated with the gauge fields themselves. Indeed, the fermion loops can be defined such that the anomalous divergence of left-handed currents involves only left-handed gauge fields. The fermion loops can also be redefined so that all vector currents are conserved and only axial-vector currents have anomalies [18]. Anomalies reflect classical symmetries that clash at the quantum level.

An intuitive perspective of fermion loop anomalies was provided by Fujikawa[19] who showed that anomalies could be viewed as resulting from the variation of the fermionic measure in a path integral formulation of quantum field theory. This view became of great importance in implementing fermions in lattice field theory and the study of the precise realization of chiral symmetry on the lattice [20].

The nonrenormalization theorem states that radiative corrections do not modify the fermion loop anomaly and that the only sources of anomalies are the small fermion loops. The Adler-Bardeen nonrenormalization theorem can be generalized to arbitrary renormalizable quantum field theories by using explicit regularization methods [21] or renormalization group arguments [22].

As discussed above anomalies can be viewed as arising from the local gauge variation of the non-local effective action describing the fermion loop dynamics. Since two gauge transformations are again a gauge transformation, the explicit functional form of anomalies is constrained by the consistency, or integrability, conditions of Wess and Zumino[23]. The full nonabelian anomaly of Bardeen[18] was shown to satisfy these consistency conditions.

Wess and Zumino[23] also showed that the consistency conditions could be integrated using an effective action involving only nonlinear realizations of Nambu-Goldstone boson fields. This effective action defines a set of Wess-Zumino terms that reproduce anomalous amplitudes for the nonabelian currents. In this case anomalies are related to the propagation of massless bosons and not fermion loops. Indeed, the chiral invariant regulator of Bell and Jackiw[14] can be viewed, in part, as generating additional Wess-Zumino terms which are then used cancel the fermion loop anomalies in their chiral invariant version of the Gell-Mann-Lévy sigma model.

Anomalies have had a remarkable impact on developments in particle physics, string theory and condensed matter physics. Classical applications to particle physics include the role of anomaly cancellation on building consistent gauge theories, the impact of anomalies on global symmetries and the constraints anomalies impose on nonperturbative dynamics in field theory.

Since anomalies reflect the quantum breaking of classical gauge symmetries, all anomalies associated with the dynamical gauge currents must cancel for the gauge dynamics to be consistent at the quantum level. Vector-like gauge theories such as electrodynamics and quantum chromodynamics are automatically free of dynamical anomalies since all of the potential anomalies can be removed through the appropriate choice of counterterms [18]. However, anomaly cancellation constrains the matter content of chiral gauge theories. Electroweak gauge theories do have chiral couplings and will generally be expected to have potential anomalies. However, all electroweak anomalies are seen to cancel in the Standard Model of particle physics[24].

Standard Model	Leptons	Quarks	Sum
$SU(2)^2 \otimes U(1)$	$-1/2$	$3 \cdot (1/6)$	0
$U(1)^3$	$1 - 1/4$	$1/36 - 8/9 + 1/9$	0

Lepton anomalies do not vanish but are canceled by the quark anomalies in each generation of fermions. A new puzzle arises: Is this remarkable cancellation an accident or a reflection of a

deeper connection between quarks and leptons such as grand unification, compositeness, or some other property of the fundamental dynamics? Anomalies continue to play a central role in building models to understand the potential for physics beyond that described by the present Standard Model.

Anomalies can also reflect a clash between global symmetries and the dynamical gauge symmetries. The electromagnetic gauge fields were seen to add an anomalous term to the divergence of the singlet axial vector current in the case of the Adler-Bell-Jackiw anomaly. Similarly, the color gauge fields of quantum chromodynamics generate an anomalous contribution to the $U(1)$ axial vector current in QCD. This anomaly provides the potential for a solution to the $U(1)$ problem of QCD: why the singlet eta' meson is not approximately degenerate with the neutral pion.

In a remarkable paper, 't Hooft[25] made a precise calculation of the effects of nonperturbative gauge fields in the form of pseudoparticles, or instantons, on anomalous symmetries. He showed that these nonperturbative effects do generate explicit breaking of the anomalous symmetries, such as baryon number violating processes in the Standard model.

Similar effects provide a possible solution of the $U(1)$ problem in QCD[26] and a large mass for the eta' meson. However, this solution also generates a new puzzle in the form of a new CP violating parameter of the strong dynamics, the theta angle, which could be of order 1 but is highly constrained by experiment to be tiny, $\theta < 10^{-9}$. This constraint on the size of the theta angle represents an outstanding fine-tuning problem of QCD and the Standard model of particle physics.

A possible solution to this fine tuning problem involves the dynamical breaking of a new Peccei-Quinn symmetry [27] that allows the theta angle to relax to zero. This dynamical breaking also introduces a new Nambu-Goldstone boson, the axion[28]. Despite the role of nonperturbative QCD dynamics, detailed predictions [29] for the phenomenology of axions could be made using anomalous current algebra with the result that electroweak scale axions could be ruled out. The strong CP problem is still outstanding for QCD and the Standard Model, and the search for an “invisible” axion continues.

As mentioned above, electroweak instanton effects associated with the electroweak gauge field can directly induce baryon number violation processes in the Standard Model. Although highly suppressed in the our present vacuum state, these effects could be important at the high temperatures present near the electroweak phase transition in the early universe. Baryon number asymmetries could also be generated via leptogenesis. In this case lepton number violating processes in the early universe can be converted to baryon asymmetries by the anomalous $B+L$ violating processes of the Standard Model.

Anomalies can also occur through the clash between purely global symmetries. While anomalies are a fundamental aspect of the short-distance dynamics, the infrared realization of the fundamental global anomalies may depend on the details of the low energy dynamics. Consistency conditions can be derived [30] to relate the infrared and ultraviolet behavior of anomalies which implies constraints on boundstate structure in composite models, anomalous couplings, etc. In QCD, the quark picture applies in ultraviolet but pions carry the QCD dynamics in infrared including the effects of the global anomalies.

The low energy theorems of current algebra can be encoded in effective field theories describing the relevant physics at low energies. Global anomalies will be reflected by specific anomalous terms in the effective Lagrangians of the low energy dynamics. Wess and Zumino showed that fermion loop anomalies could be reproduced using specific nonlinear realizations of the global current algebra and encoded in Wess-Zumino terms [31]. In the case of chiral symmetry, the full set of Wess-Zumino terms predicts additional anomalous multipion interactions including the anomalous coupling of pions to external gauge fields and currents. An elegant formulation of these anomalous interactions, including the coupling to gauge fields, was made by Witten[32] and extended by others [33].

Anomalies can also be discussed within the context of the local symmetries associated with gravity and external gravitational fields. Kimura[34] showed that U(1) axial-vector anomaly contains an additional contribution in the presence of background gravitational fields.

$$\partial^\mu J_{5\mu} = \frac{1}{768\pi^2} \epsilon_{\mu\nu\sigma\tau} R^{\mu\nu\alpha\beta} R_{\alpha\beta}^{\sigma\tau}$$

This contribution is related to a topological index for the gravitational field, similar to the gauge anomaly where the anomalous divergence is related to a topological index for the nonabelian gauge field. The consistency of the U(1) gauge interactions in the Standard Model implies that the sum of all contributions of the individual fermions to the gravitational anomaly must cancel. Another remarkable feature of the Standard Model is that this cancellation, in fact, automatically occurs within each generation.

Standard Model	Leptons	Quarks	Sum
$R^2 \otimes U(1)$	$2(-1/2) + 1$	$3(1/3) + 3(-2/3) + 6(1/6)$	0

In this case, the lepton and quark gravitational anomalies cancel independently. Can this fact provide any additional hints for the specific origin of the fermion representations used by nature?

Pure gravitational anomalies also exist in 2, 6 and 10 dimensions [35]. These anomalies could imply the existence of possible obstructions to the formulation consistent gravitational couplings in

these dimensions. Gravity has many local symmetries including general coordinate invariance and the local Lorentz invariance of the tangent space required to define fermions. The consistency of gravitational anomalies and the connection between anomalies associated with general coordinate symmetries and the gauging of the local Lorentz symmetries can be derived [36].

Anomalies have also played an important role in the development of string theory. String theories must avoid both gauge and gravitational anomalies. Anomalies can be seen by studying the massless sector of the theory as the anomalous low energy theorems require a nonanalyticity of the relevant amplitudes. For example, the massless sector of open string states can contribute to anomalous amplitudes at one loop where the relevant terms are described by a cylinder with a large radius but small height. The cylinder function has another infrared limit where a closed string tube connects two disks where the tube has a small radius and large length. In this limit, the anomalous behavior arises at tree level from massless poles in the gravity sector of the theory. Green and Schwarz[37] made the crucial observation that loop anomalies found in the open string sector could cancel against the tree level anomalous couplings of gravitational sector with both contributions arising from the same cylinder contribution. With this observation modern string theory was born. It is amusing to observe that this mechanism for the cancellation of string anomalies is similar to the anomaly cancellation seen by Bell and Jackiw in their chiral invariant regularization of the Gell-Mann-Lévy sigma model or in the Standard Model between pions and lepton loops.

Conclusions

Sakata's detailed study of mesotron decays led him to speculate about the decay of the neutral mesotron to photons. New technologies in quantum field theory allowed the precise calculation of the neutral pion decay amplitudes following the ideas suggested by Sakata but left puzzles concerning the discrepancies between pseudoscalar and axial-vector couplings of the pion.

The suggestion of a hidden chiral symmetry of the strong interactions focused attention on the role of the axial-vector current and a new realization of symmetries in the form of PCAC with the role of the pion as a Nambu-Goldstone boson. The extension of PCAC to amplitudes involving photons led to the discovery of anomalies and a more fundamental understanding of quantum field theories and their symmetries.

The discovery and analysis of the complete nonabelian anomaly coupled with nonrenormalization theorems showed that anomalies reflect very fundamental aspects of quantum field theory. Anomalies have a remarkably broad impact on theoretical particle physics, from anomaly cancel-

lation as an intrinsic part of model building to determining subtle aspects of realizations of global symmetries and constraints on the nonperturbative dynamics. Anomalies also play an important role in modern string theory and can be expected to be an essential part of our ultimate description of nature.

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